Astrodynamics 101

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Definitions and Purpose

Astrodynamics

Orbital Mechanics

} **Satellite and Missile Dynamics**

Celestial Mechanics → **Planetary Object Dynamics**

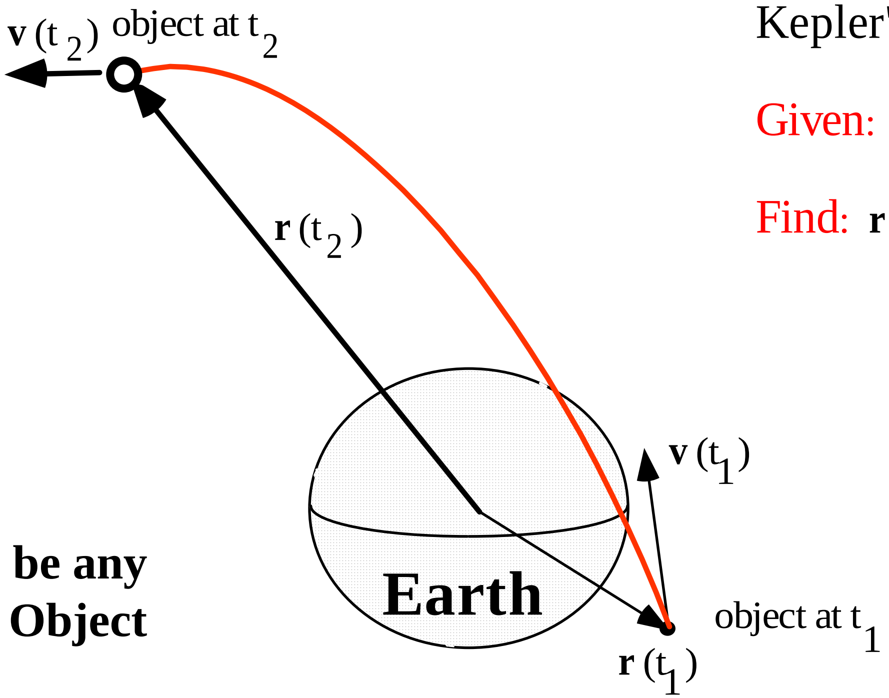
Astrodynamics is an exercise of trajectory computation for space objects, either analytically or numerically with the highest <u>speed</u>, <u>accuracy</u> and <u>robustness</u> whenever possible

The Astrodynamics 101 Problem

The Important Kepler's Problem

Given: **Equations of Motion and Initial State Vector**

To Find: **Final State Vector (position, velocity, . .)**



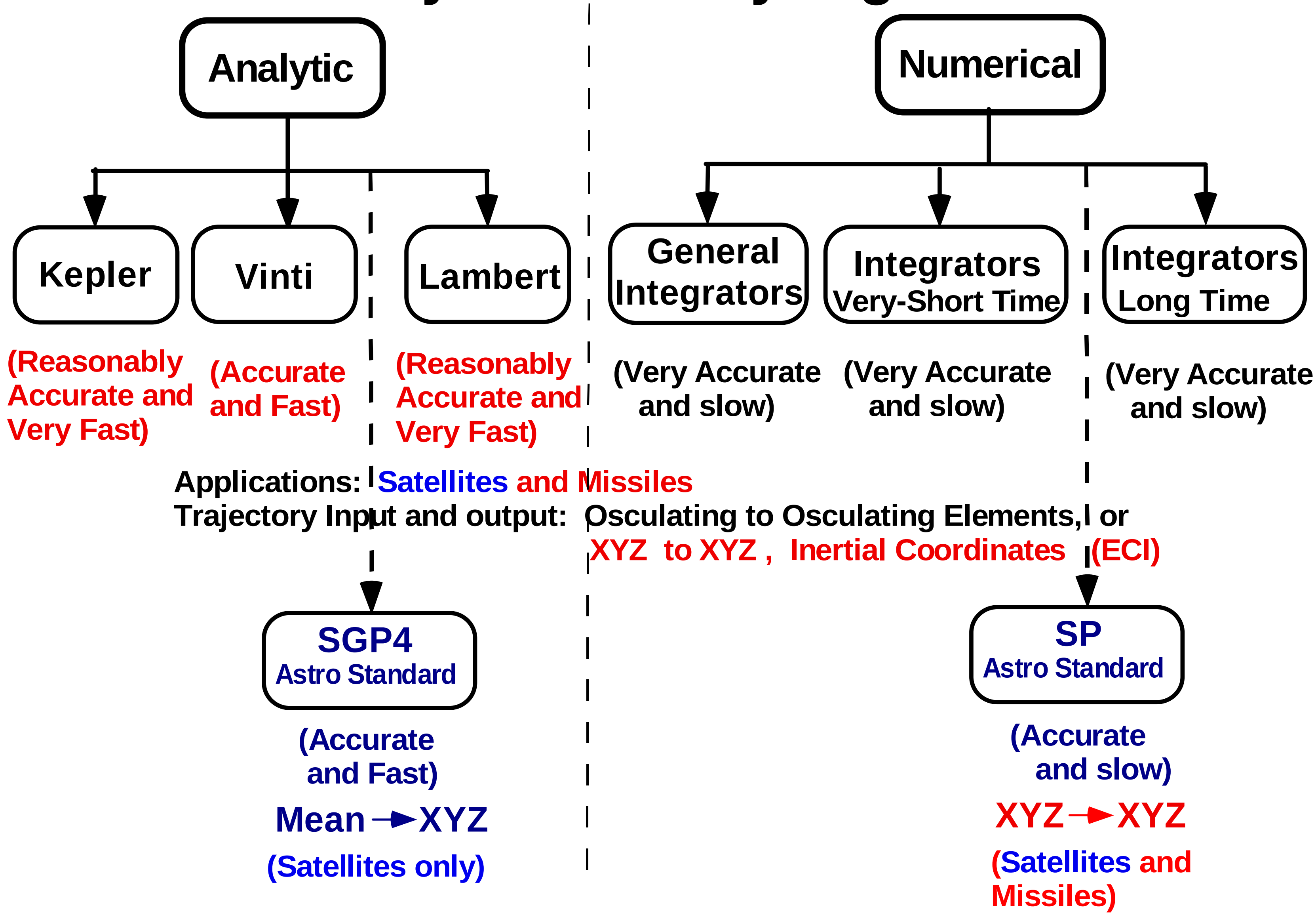
Kepler's Problem

Given: $\mathbf{r}(t_1), \mathbf{v}(t_1), t_1, t_2$

Find: $\mathbf{r}(t_2), \mathbf{v}(t_2)$

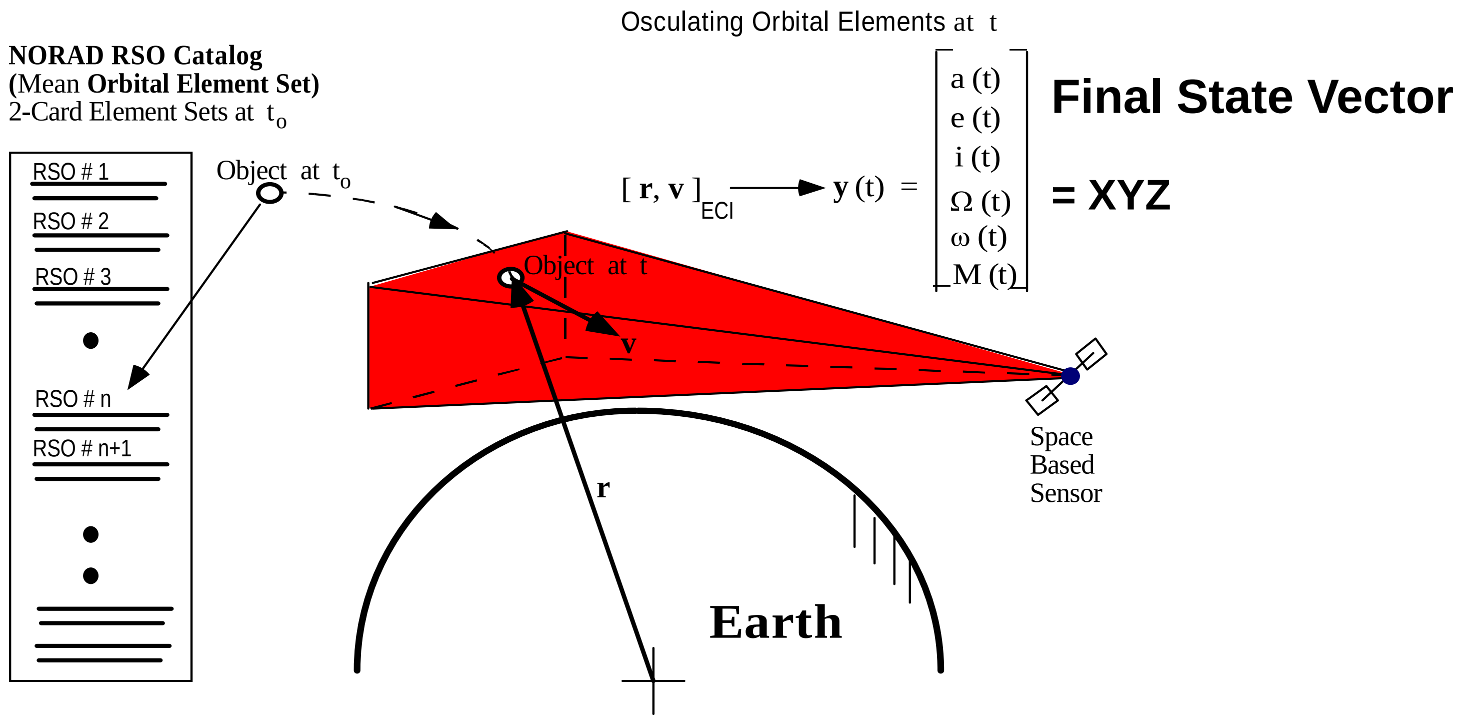
Earth can be any Planetary Object

Astrodynamics Key Algorithms



Satellite Trajectory Propagation (1)

(Analytic Method, SGP4)



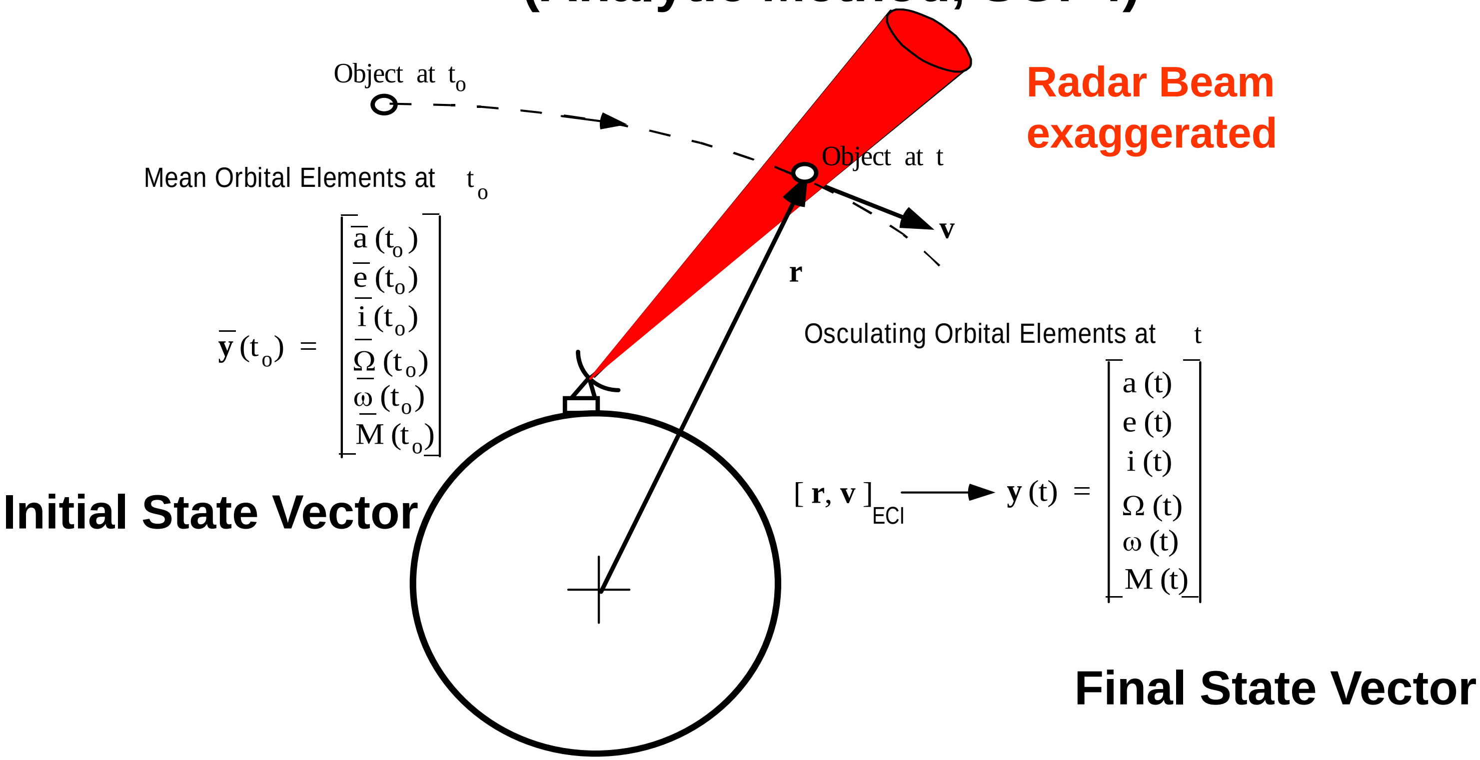
Initial State Vector

Given: Mean Orbital Elements at t_o , $\bar{\mathbf{y}}(t_o)$

Find: Osculating Orbital Elements at t, $\mathbf{y}(t) = [\,\mathbf{r}, \mathbf{v}\,]_{ECI}$ **= XYZ or 3D Coordinate System**

Astro-Standard, SGP4 Prediction of position and velocity at t has a Different Format compared to input at t_o

Satellite Trajectory Propagation (2)
(Analytic Method, SGP4)



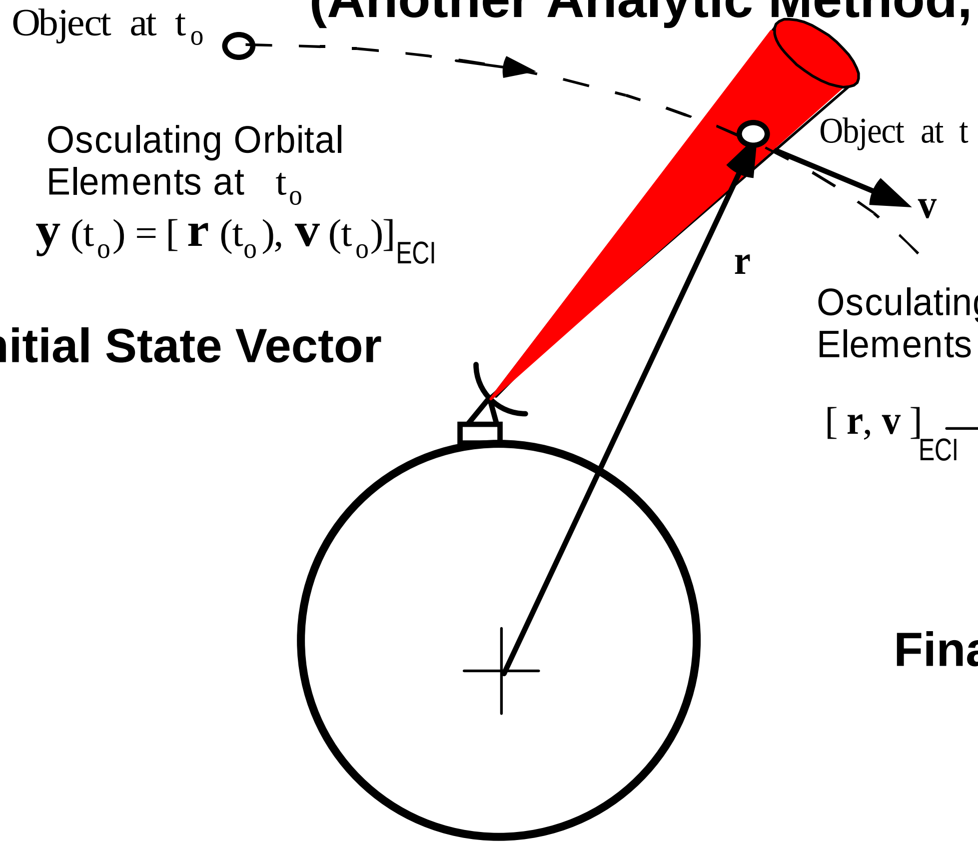
Given: Mean Orbital Elements at t_o, $\bar{\mathbf{y}}(t_o)$

Find: Osculating Orbital Elements at t, $\mathbf{y}(t) = [\mathbf{r}, \mathbf{v}]_{ECI}$ **= XYZ or 3D Coordinate System**

Differences in Input and Output Formats of the SGP4 cause problems for implementing more accurate Satellite Tasking

Satellite Trajectory Propagation (3)

(Another Analytic Method, Vinti)



Osculating Orbital Elements at t_o

$\mathbf{y}(t_o) = [\,\mathbf{r}(t_o), \mathbf{v}(t_o)]_{ECI}$

Initial State Vector

Osculating Orbital Elements at t

$[\,\mathbf{r}, \mathbf{v}\,]_{ECI} \longrightarrow \mathbf{y}(t) = \begin{bmatrix} a(t) \\ e(t) \\ i(t) \\ \Omega(t) \\ \omega(t) \\ M(t) \end{bmatrix}$

Final State Vector

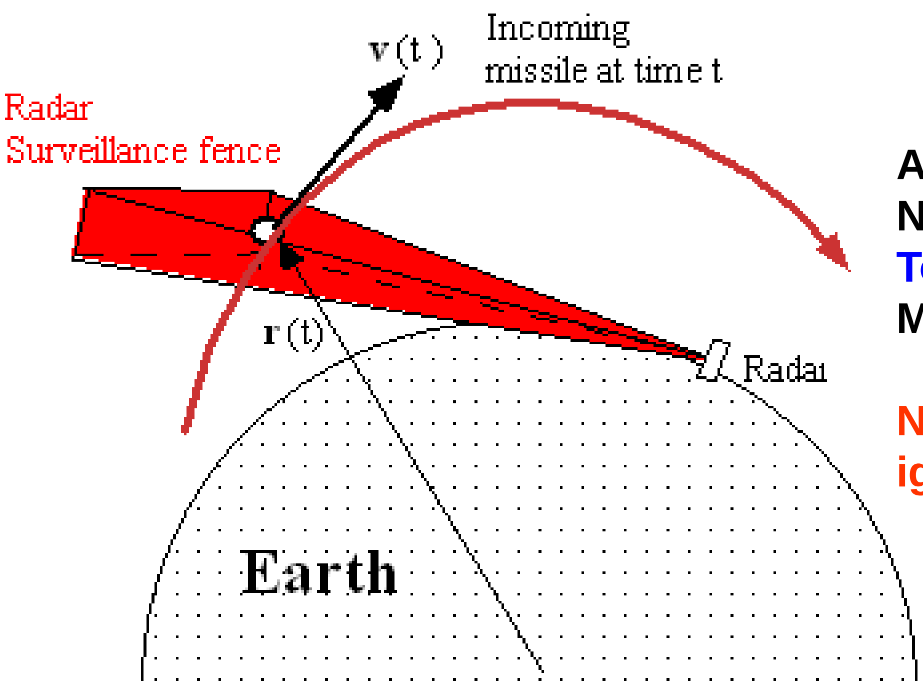
Given: Osculating Orbital Elements at t_o, $\mathbf{y}(t_o)$ **= XYZ or 3D Coordinate System**

Find: Osculating Orbital Elements at t, $\mathbf{y}(t) = [\,\mathbf{r}, \mathbf{v}\,]_{ECI}$

Non-Astro-Standard, Vinti Prediction of position and velocity at t has the Same Format compared to input at t_o, same as SP

Missile Trajectory Propagation

(A Ground Based Radar)



All Radars use
Numerical Methods
To Calculate
Missile Trajectories

No Analytic Method due to ignorance of the Vinti predictor

To find: y (t) = [r (t), v (t)] **= XYZ or 3-D Coordinate System**

Analytic Method: Vinti
Numerical Method: SP (Special Perturbation)
Note: Input and Output are in the Same Format

Astrodynamics Objectives

Fast, Accurate, Robust

Compute trajectories of satellites, missiles and space objects as Fast, Accurate and Robust as possible

Mathematical Physics Interpretation

Astrodynamics: Understanding the Force

(Newton's Formula = Equations of Motion)

Mass • Acceleration = Force

General Formula:

$$\frac{d^2 \mathbf{r}}{d t^2} = \mathbf{f}\left(t,\ \mathbf{r},\ \frac{d\mathbf{r}}{dt}\right) = \mathbf{F}$$

Orbiting Satellites:

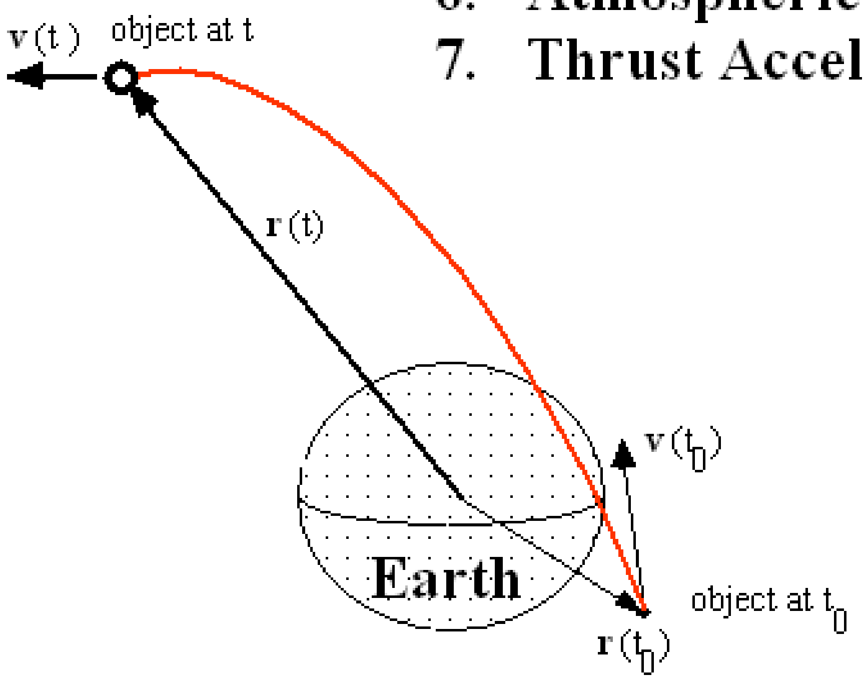
$$\frac{d^2 \mathbf{r}}{d t^2} = \mathbf{f}\left(\frac{\mathbf{r}}{r^3}\right) = \mathbf{F}_{gravity}$$

Missiles / Aircraft:

$$\frac{d^2 \mathbf{r}}{d t^2} = \mathbf{F}_{gravity} + \mathbf{F}_{thrust} + \mathbf{F}_{aero} + \mathbf{F}_{wind}$$

Forces on a Satellite and Missile (1)

1. Earth central gravity (low)
2. Earth Oblate spheroidal gravity (high)
3. Sun gravity
4. Moon gravity
5. Solar Radiation Pressure
6. Atmospheric (Air) Drag
7. Thrust Acceleration



Given: Equations of Motion

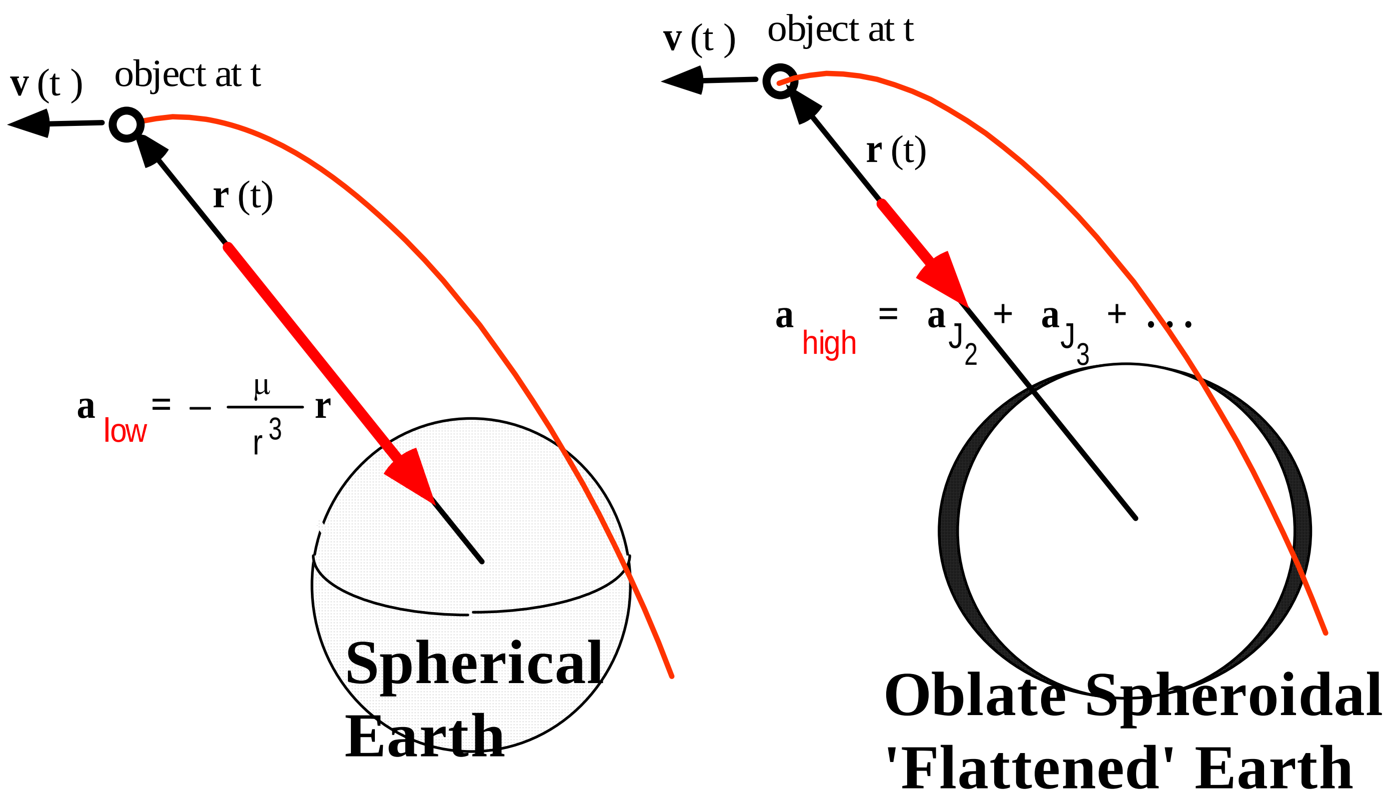
Kepler's Problem

Given: $\mathbf{r}(t_0)$, $\mathbf{v}(t_0)$, t_0, t

Find: $\mathbf{r}(t)$, $\mathbf{v}(t)$

Accelerations on a Satellite and Missile (2)

1. **Earth central gravity (low)**
2. **Earth Oblate spheroidal gravity (high)**



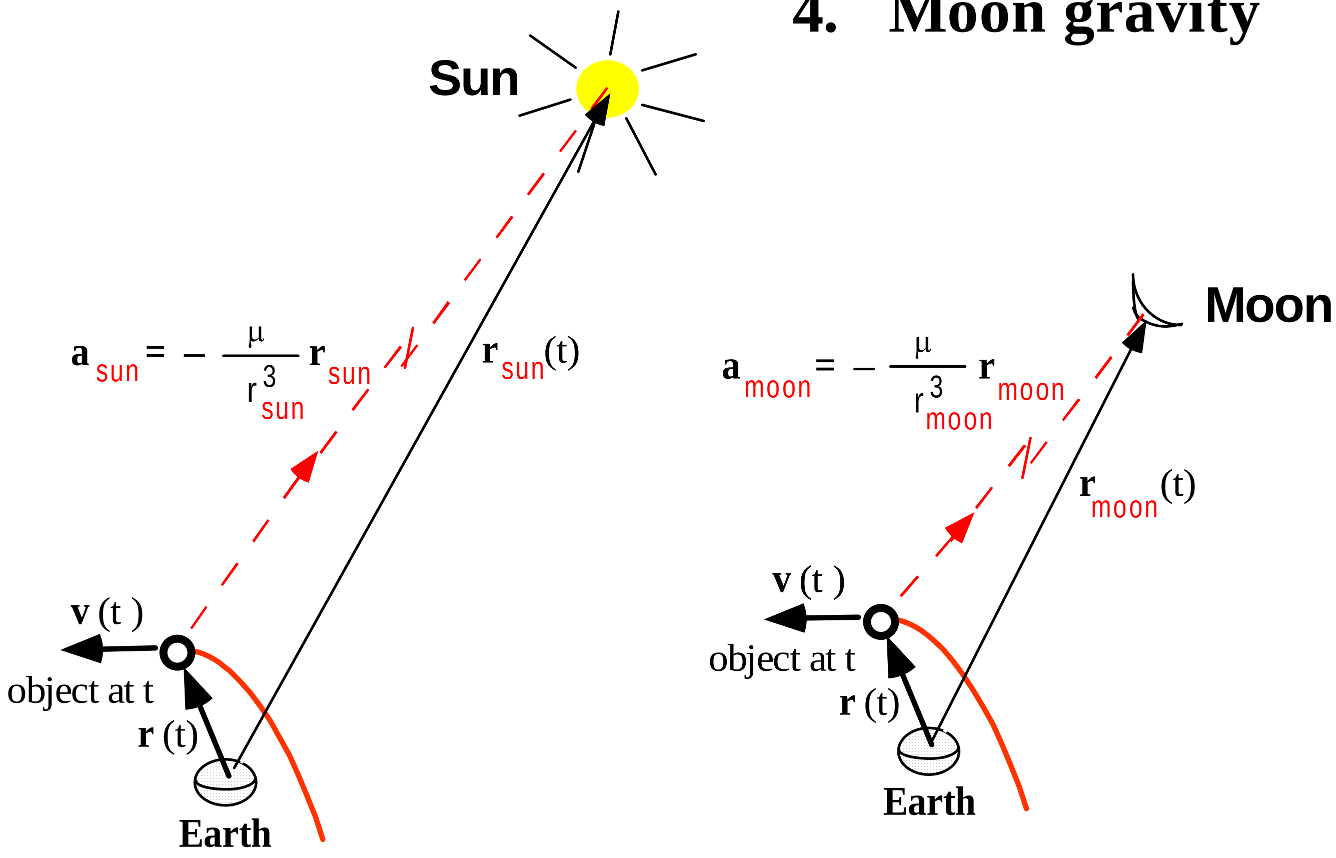
Example:
GEO object at t
with r = 40,000 km

$|\mathbf{a}_{low}| = 0.2\ \text{g}$

$|\mathbf{a}_{high}| = 0.8 \times 10^{-6}\ \text{g}$

Accelerations on a Satellite and Missile (3)

3. **Sun gravity**
4. **Moon gravity**



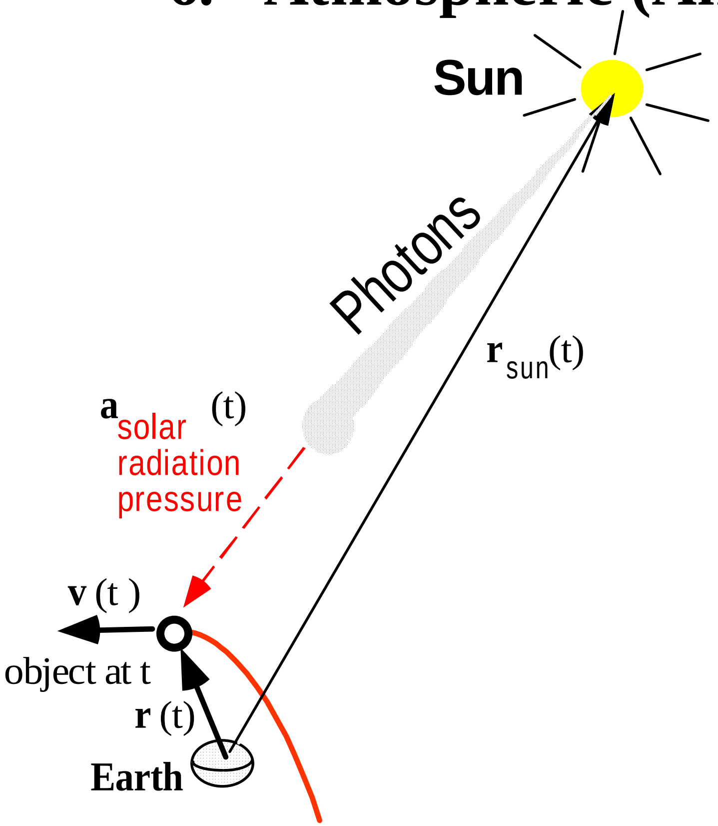
Example:
GEO object at t
with r = 40,000 km

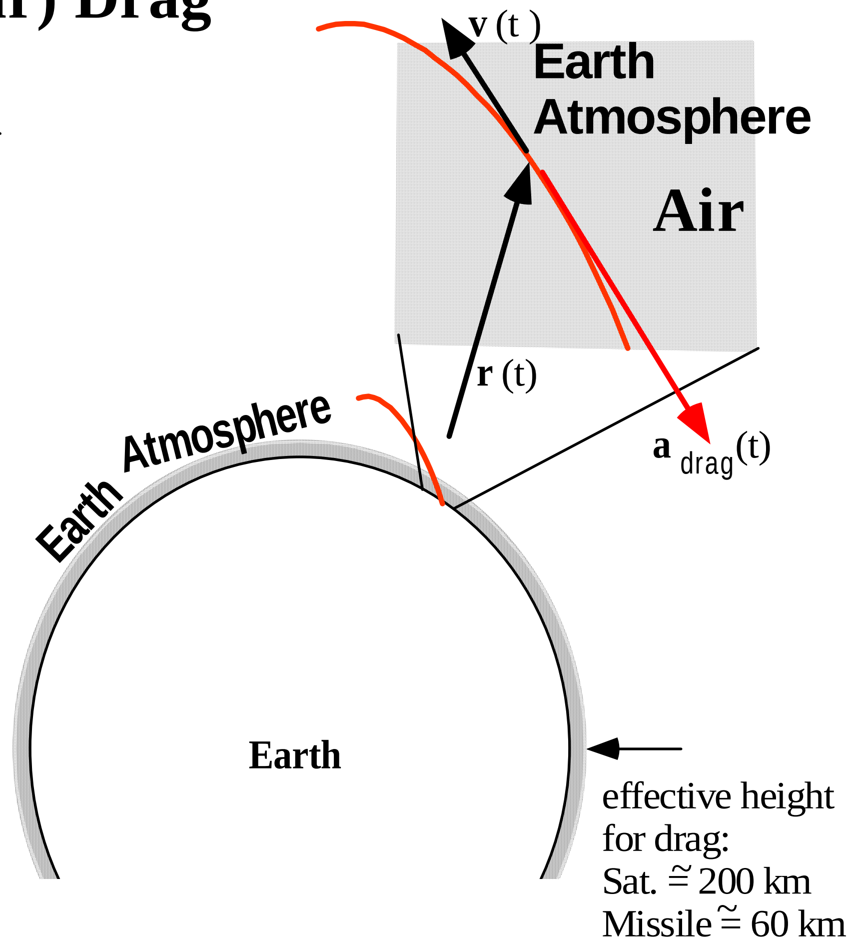
$|\mathbf{a}_{sun}| = 0.2 \times 10^{-6}$ g $\quad$ $|\mathbf{a}_{moon}| = 0.5 \times 10^{-6}$ g

Accelerations on a Satellite and Missile (4)

5. Solar Radiation Pressure

6. Atmospheric (Air) Drag





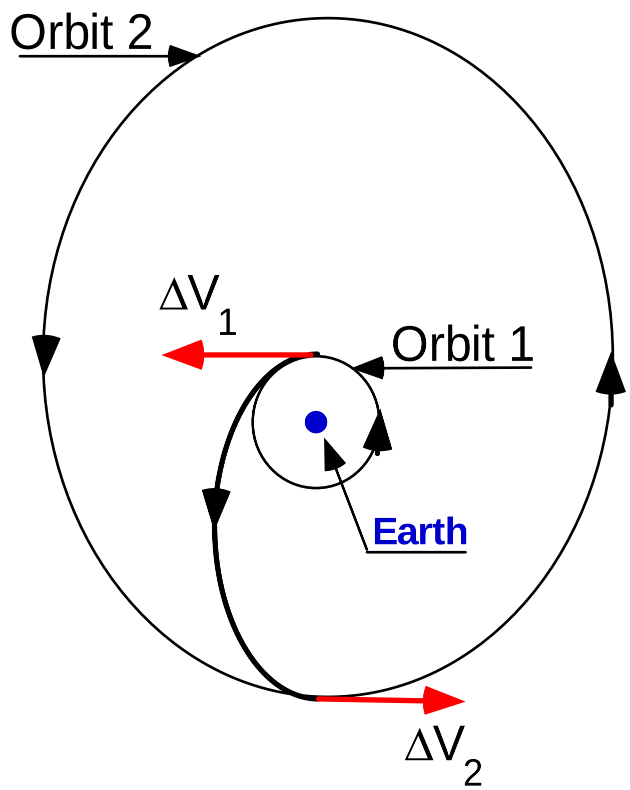
Example:
GEO object at t
with r = 40,000 km

$$\left| \mathbf{a}_{\text{solar radiation pressure}} \right| = 0.3 \times 10^{-8}\ \text{g}$$

$$\left| \mathbf{a}_{\text{drag}} \right| = 0\ \text{g}$$

Accelerations on a Satellite and Missile (5)

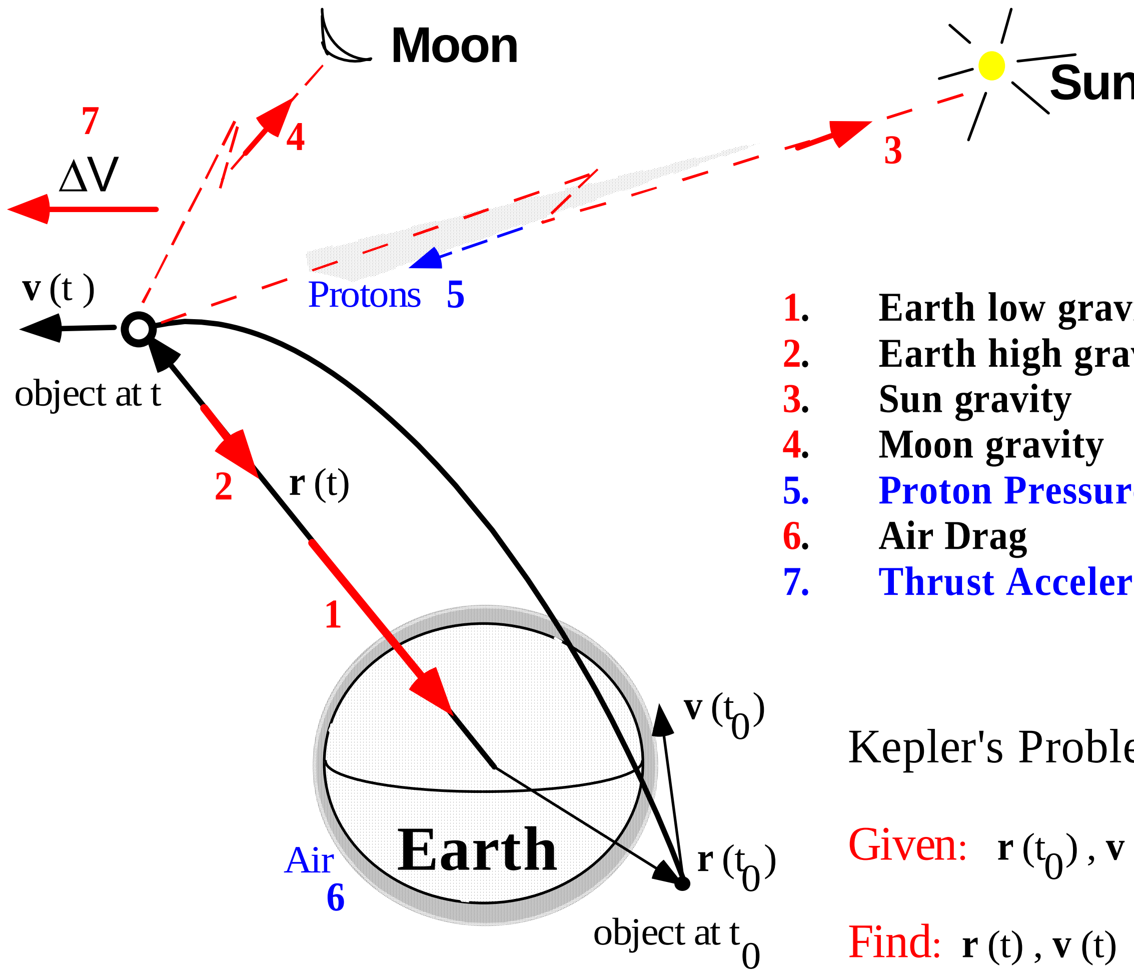
7. Thrust Acceleration



Satellite Thrusting
(Orbit Change or
Satellite Maneuver)

Missile Thrusting
(Ascending from Earth)

Summary, Forces on a Satellite and Missile (6)



1. **Earth low gravity (Pull)**
2. **Earth high gravity (Pull)**
3. **Sun gravity (Pull)**
4. **Moon gravity (Pull)**
5. **Proton Pressure (Push)**
6. **Air Drag (Pull)**
7. **Thrust Acceleration (Push)**

Kepler's Problem

Given: $\mathbf{r}(t_0)$, $\mathbf{v}(t_0)$, t_0, t

Find: $\mathbf{r}(t)$, $\mathbf{v}(t)$

Forces in The Equations of Motion

Equations of Motion

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{a}_d$$

$-\frac{\mu}{r^3}\mathbf{r}$ = low gravity
Force 1

$\mathbf{a}_d$ = disturbed acceleration
Forces (2 to 7)

Analytic Methods

$\mathbf{a}_d = 0$

Kepler
Newton

Keplerian Trajectories
(Fast, quite accurate)

$|\mathbf{a}_d| \ll \left|-\frac{\mu}{r^3}\mathbf{r}\right|$

SGP4
Vinti

Non-Keplerian Trajectories
(Fast, accurate)

Numerical Methods

$\mathbf{a}_d$ = function (J_2 , J_3 , J_4 , Drag , . . .)

Encke
Cowell

Non-Keplerian Trajectories
(Slow, very accurate)

$\mathbf{a}_d$ is arbitrary

Proper Use of Analytic and Numerical Methods to compute trajectories enables efficient processing for Satellite and Missile

Astrodynamics Objectives

Small Problems:

1. How fast is fast?   2. How accurate is accurate?

Fast and Accurate

O (micro-second or 10^{-6} s)   O (millimeter or 10^{-6} km)

Compute one trajectory of a satellite or missile by a 3GHz PC

3. How robust? An algorithm should not break down under extreme conditions

Analytic Prediction

What Astrodynamics does:

Predict the trajectory of a space object

= given (r_1, v_1, t_1, t_2); find (r_2, v_2)

Kepler: **Very fast and reasonably accurate (if you know the limitations)**

Vinti: **Fast and accurate** (J_2, J_3, ~70% J_4)

Predict the velocities of a space object

= given (r_1, r_2, t_1, t_2); find (v_1, v_2)

Lambert: **Very fast and reasonably accurate**

Numerical Integration

What Astrodynamics Does:

Propagate trajectory parameters of a space object given (r, v, a, . . . , σ_r , σ_v , . .)

Getting a basic solution is easy, use slow but accurate methods of Numerical Integration!

Challenging Problems

The Problem:

1. An Analytic Solution is Fast but Slightly Inaccurate

2. A Numerical Solution is Slow but Accurate

The Challenge:

To find a Fast and Accurate analytic solution

Currently (2010), how fast is fast? (3 GHz PC, One 2000 s trajectory)

- **An Analytic Solution (~ 1 micro-second, $1\text{x}10^{-6}$ s)**
- **A Numerical Solution (milli-second, time dep, $1\text{x}10^{-3}$ s)**

Payoffs for Overcoming the Challenge

Major Payoffs:

1: Accurate and Fast missile trajectory prediction
=> Cheaper interceptor without an expensive homing sensor

2: Accurate and Fast satellite trajectory prediction
=> Fast prediction of huge number of space objects days ahead in order to reduce accidental collisions (Less debris, less satellite tracking radar maintenance)

The Challenge:
To find a Fast and Accurate analytic solution