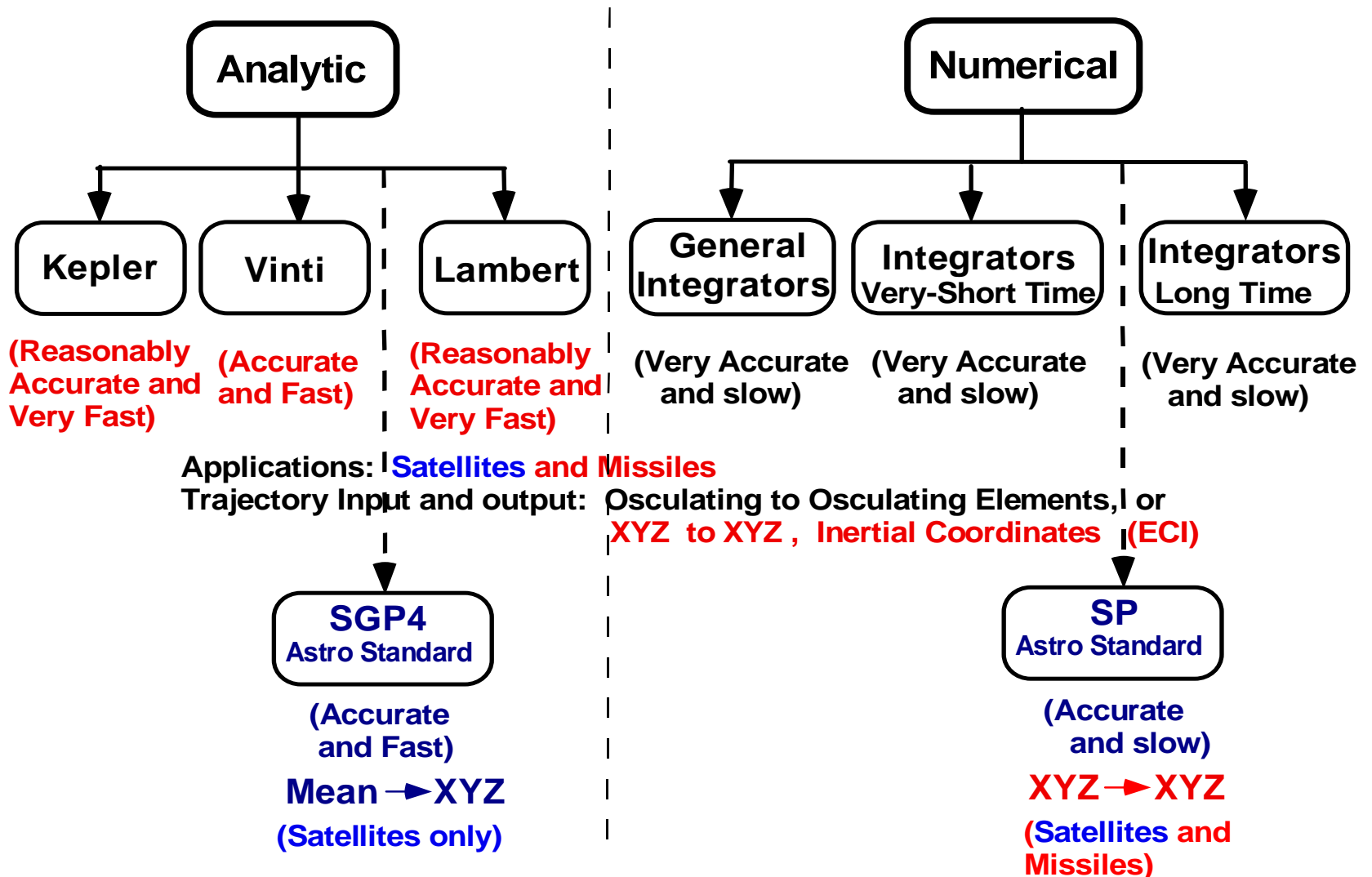

SGP4 vs. Vinti

**Gim Der
(DerAstrodynamics)**

Feb 23, 2010

Astrodynamics Key Algorithms



Comparison of Analytic Methods

	Kepler	SGP4 Astro Standard	Vinti
<i>Theory</i>	Simple formulation	Complicated formulation (Averaging technique)	Elegant formulation (Physics + Geometry)
<i>History</i>	~ 400 years (theory) ~ 200 years (algorithm) ~ 50+ years (working computer program)	~ 60 years (theory) ~ 50 years (algorithm) ~ 40+ years (working computer program)	~ 60 years (theory) ~ 50 years (algorithm) ~ 10+ years (working computer program)
<i>Application</i>	Satellites and missiles	Satellites (elliptic orbits)	Satellites and missiles
<i>Problem?</i>	No singularity	Some singularities	No singularity

People Behind Analytic Methods

Kepler

Tycho Brahe
Johann Kepler
Issac Newton
(1600 ~ 1700)

Too many
computer programs
and key
developers

Newtonian
Mechaincs
(second-order
differential eqns
of motion)

SGP4
Astro Standard

von Zeipel
Delaunay
Kozai
Brouwer (~ 1950)

Too many computer
program developers
Gov. financed

Newtonian
Mechaincs
(second-order
differential eqns
of motion)

Vinti

Hamilton
Jacobi
Laplace
Vinti (~ 1950)

Key computer
program developer
Gim Der
Self financed

Hamiltonian
Mechanics
(first-order
differential eqns
of motion)

Two Key Analytic Methods

Comparison: Capability and Cost

SGP4 Astro Standard

1. Not work for Missiles
2. Satellite Orbits (circle, Ellipse)
3. Singularities at
 $i = 0, 63.4, 90$; $e = 0$.
4. Initialization difficulty
(input mean orbital elements,
output inertial state vector)
5. Need \$\$\$ maintenance
(fixes for new singularities,
many versions/updates)

Vinti Theory / Der Code

1. Works for Missiles
2. Satellite Orbits (circle, Ellipse,
parabola, hyperbola)
3. No singularities for
any satellite orbits
4. No initialization difficulty
(same I/O format in
inertial state vectors)
5. Need very little maintenance
(Code completely tested for:
missiles in 1998, satellites in 2006)

Similar Computational Speed and Accuracy

Applications

SGP4:

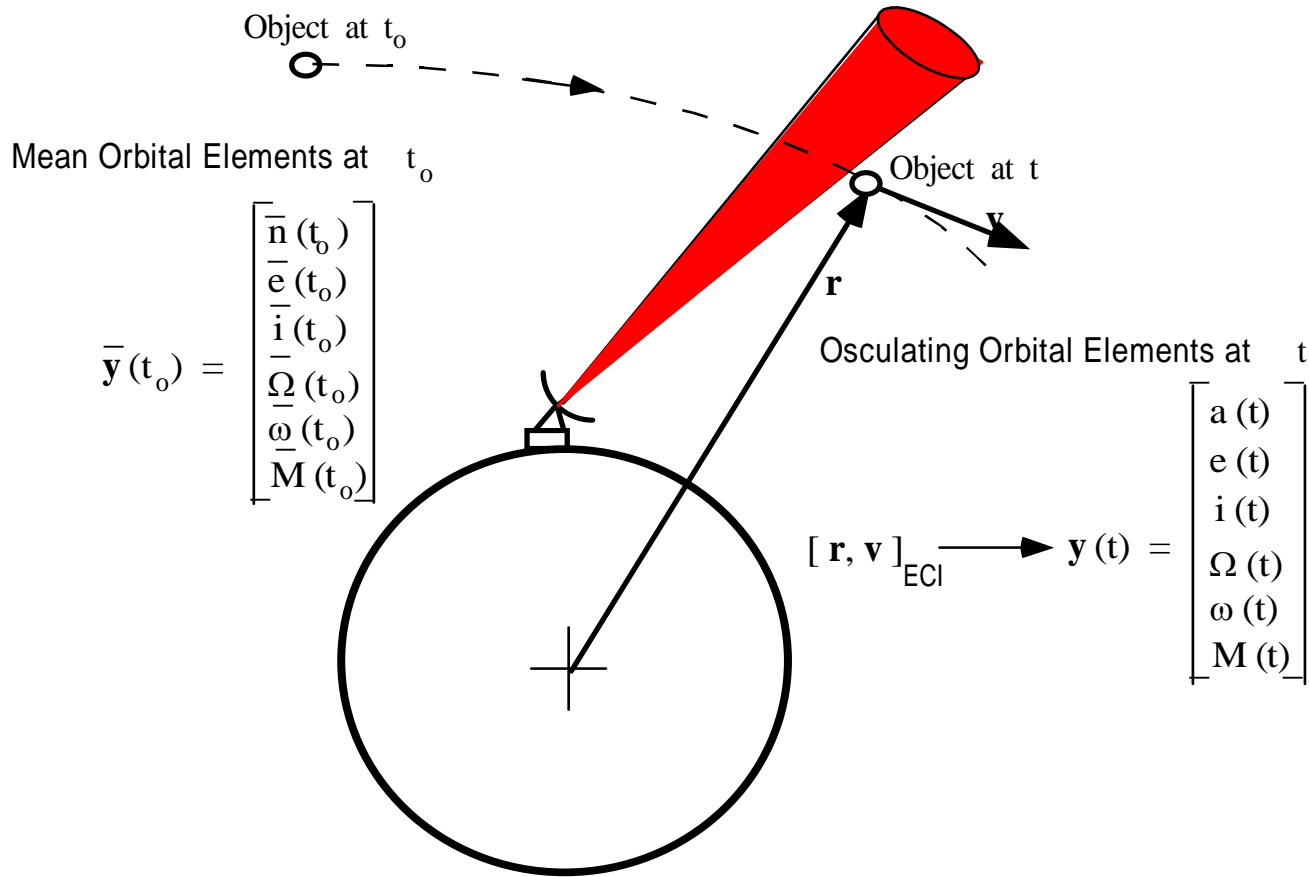
- **US StratCom To Manage Satellite, Space Debris Tracking**
- **US Radar pointing algorithm**
- **Implementing SP Tasking Difficult due to I/O Format Problem**

Vinti:

- **US StratCom To Manage Satellite, Space Debris Tracking**
- **US Radar pointing algorithm**
- **Implementing SP Tasking Easy due to Same I/O Format**
- **Collision Prediction for Objects with only tracked ECI state vectors (unknown space objects, missiles)**
- **Radar observation data IOD**
- **Ballistic Launch and Impact Point Predictions**
- **Ballistic Missile Intercept Guidance**

Satellite Look Angle/Search by SGP4

Difficulty of conversion from ECI vector to Mean Element Set

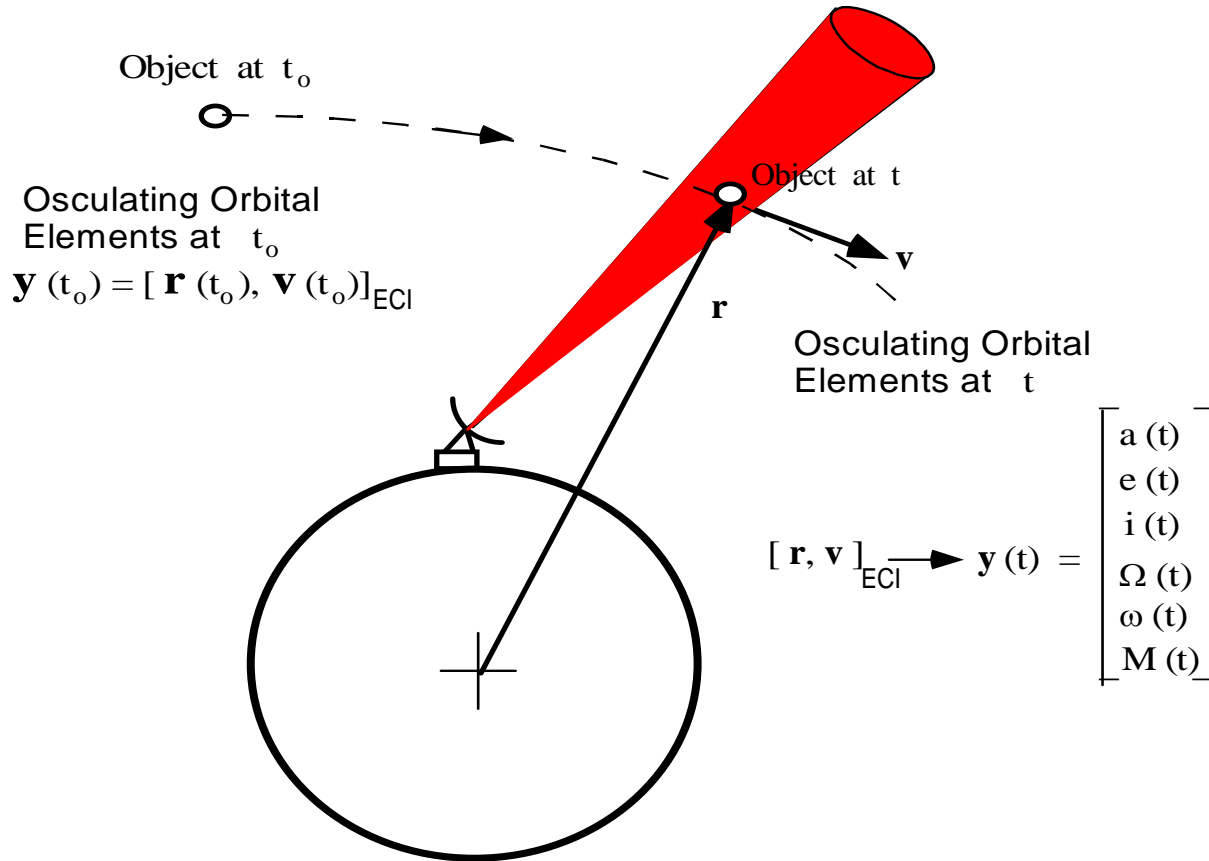


Given: Mean Orbital Elements at t_0 , $\bar{\mathbf{y}}(t_0)$ = 2-Card Element Set

Find: Osculating Orbital Elements at t , $\mathbf{y}(t) = [\mathbf{r}, \mathbf{v}]_{\text{ECI}}$ = XYZ or Inertial State Vector Coordinates

Satellite Look Angle/Search/Track by Vinti/Der

Same Input/Output leads to Simple Applications

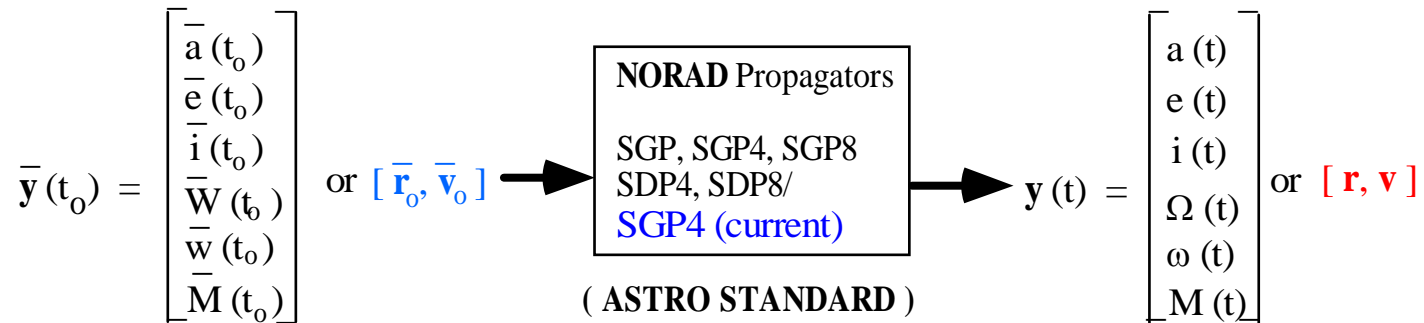


Given: Osculating Orbital Elements at t_0 , $\mathbf{y}(t_0)$

Find: Osculating Orbital Elements at t , $\mathbf{y}(t) = [\mathbf{r}, \mathbf{v}]_{\text{ECI}} = \text{XYZ or Inertial State Vector Coordinates}$

Analytic Forward Propagation “Mean Elements / XYZ to Osculating Elements”

Some singularities, different I/O formats



Mean Elements to Osculating Elements / **XYZ**

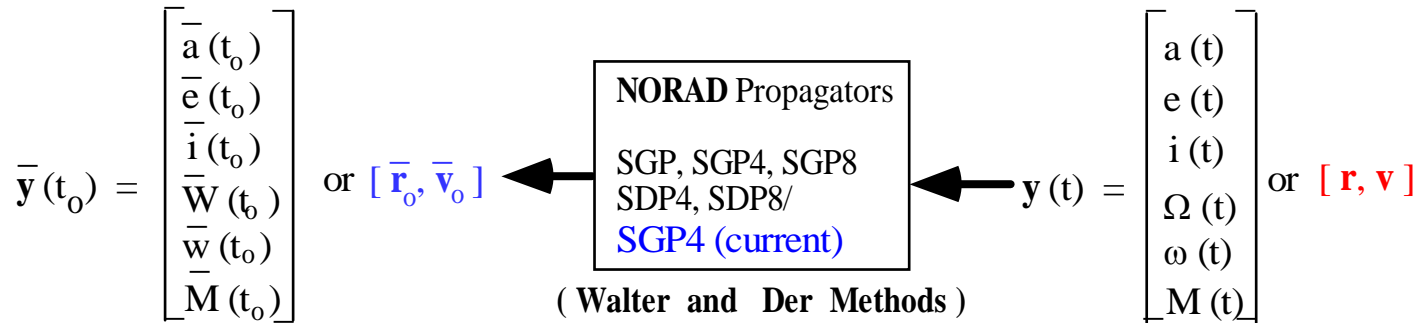
No singularities, same I/O formats



XYZ / Osculating Elements to Osculating Elements / XYZ

Analytic Backward Propagation “Osculating Elements / XYZ to Mean Elements”

Not Straight forward, sometimes not possible



Osculating Elements / XYZ to Mean Elements

Straight forward, always possible

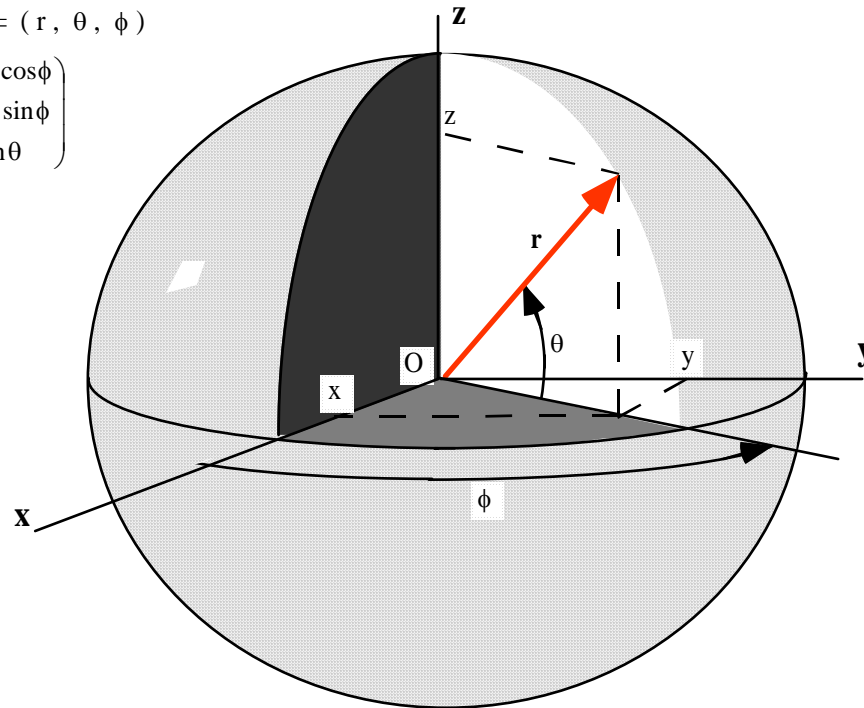


XYZ / Osculating Elements to Osculating Elements / XYZ

Kepler Method

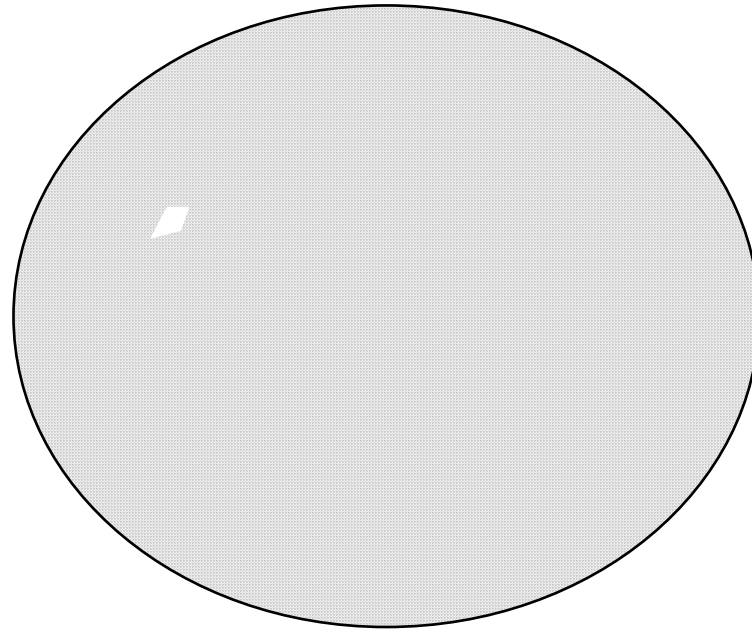
Spherical coordinates = (r, θ, ϕ)

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cos\theta \cos\phi \\ r \cos\theta \sin\phi \\ r \sin\theta \end{pmatrix}$$



Kepler method assumes a **Spherical Central Body**
(Spherical Earth or Sun)

Brouwer Method (SGP4) (No Physics or Universal Variable Formulation)



Brouwer method assumes a **Ellipsoidal Centual Body**.
The theory, which does not employ any PHYSICS
formulation, uses **averaging** techniques, and therefore
the orbital elements are called **MEAN** orbital elements.

Vinti Method (Physics and Universal Variable Formulation)

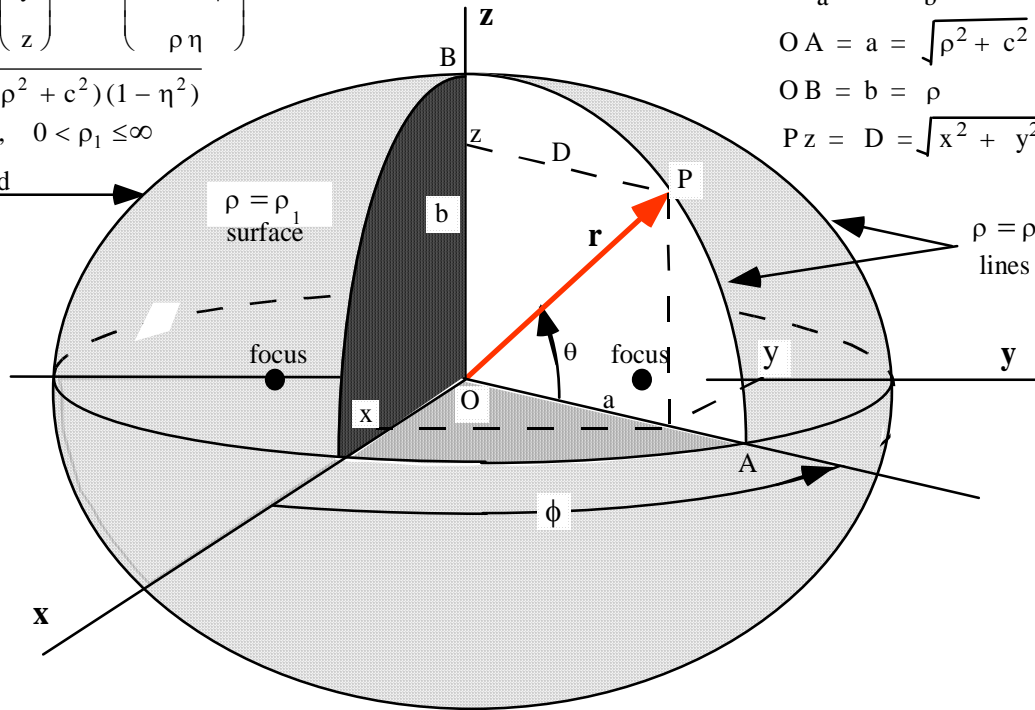
Oblate spheroidal coordinates = (ρ, η, ϕ)

$$\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} D \cos \phi \\ D \sin \phi \\ \rho \eta \end{pmatrix}$$

$$D = \sqrt{(\rho^2 + c^2)(1 - \eta^2)}$$

$$\rho = \rho_1, \quad 0 < \rho_1 \leq \infty$$

spheroid



Equation of an oblate spheroid of semi-axes a and b

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1$$

$$OA = a = \sqrt{\rho^2 + c^2}$$

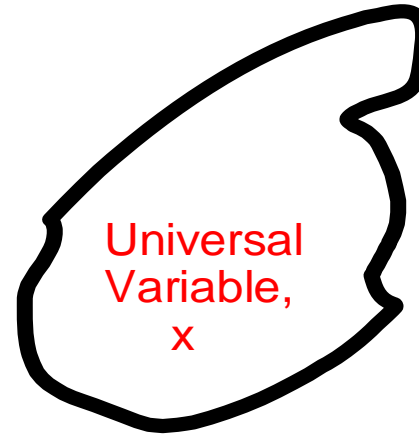
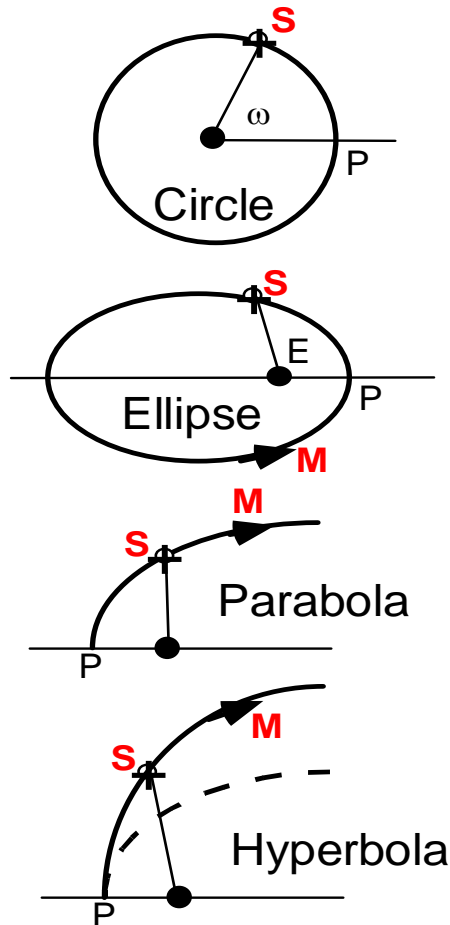
$$OB = b = \rho$$

$$Pz = D = \sqrt{x^2 + y^2}$$

Vinti method assumes a **Oblate Spheroidal Centual Body** (Earth or any body). The theory an elogent employs **PHYSICS** formulation. Since the input/output format is the same as those of the Kepler and numerical methods the computation is straight forward and accurate.

1st Step to Vinti/Der's Code

Understand the **Universal Variable, x**



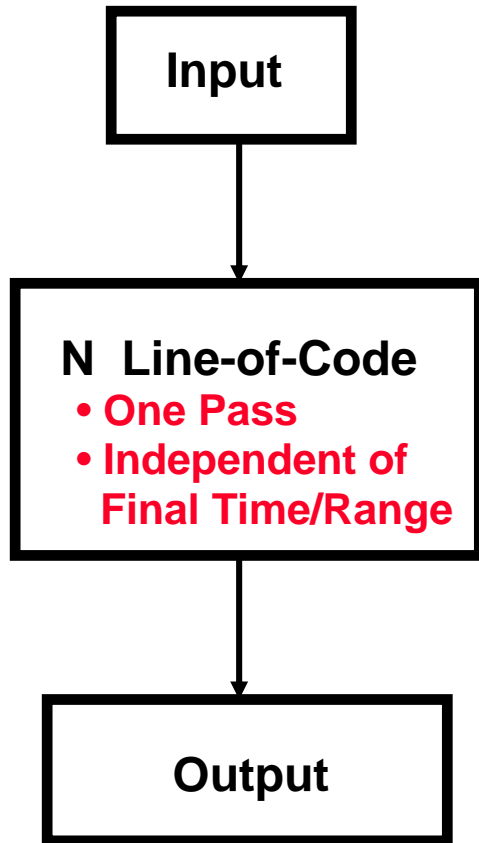
The value of x automatically adjusts to all four orbits and seamlessly transitions from one orbit to another

P = perigee,
S = satellite, **M** = missile
(Vinti can compute both)

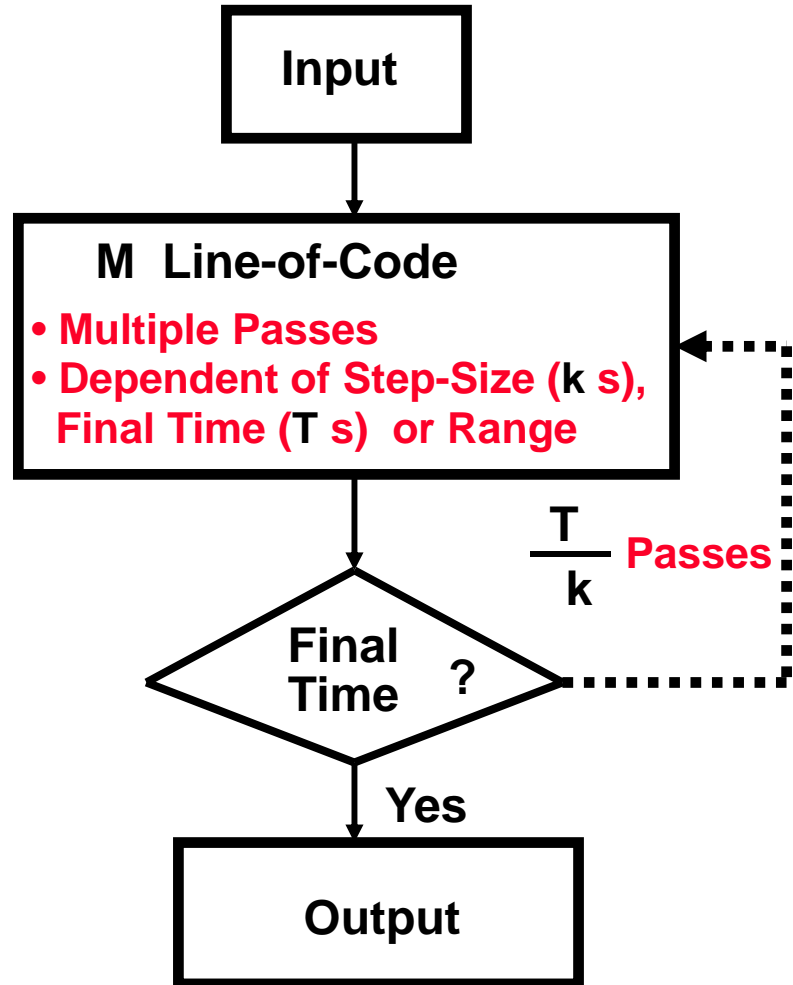
Analytic vs Numerical Methods

Computational Speed and Accuracy

Analytic Method **Speed**



Numerical Method **Accuracy**



Vinti History

- **Vinti's Theory, Originated in 1959, Improved in 1966**
Professional Jealousy and Political Fiddling,
See Interesting Reading: [Brouwer vs Vinti](#)
- **Earlier Problematic Code, Developed from 1960 to 1980 by**
Ford Aerospace, TRW (MinuteMan), Goddard, NSA
- **Code developed and Copyrighted by Gim Der in 1998**
(Published in Vinti Book by AIAA, "Orbital & Celestial Mechanics")
- **Possible Primary Applications**
 1. **US StratCom To Manage Satellite, Space Debris Tracking**
 2. **US SENSOR pointing algorithm**
 3. **Easy Implementation of SP Tasking due to Same I/O Format**
 4. **Collision Prediction for Objects with only tracked ECI state vectors (unknown space objects, missiles)**
 4. **Radar observation data IOD**
 6. **Ballistic Launch and Impact Point Predictions**
 7. **Ballistic Missile Intercept Guidance**

Analytic and Numerical Methods

Ballistic Missile Impact Point Prediction (IPP)

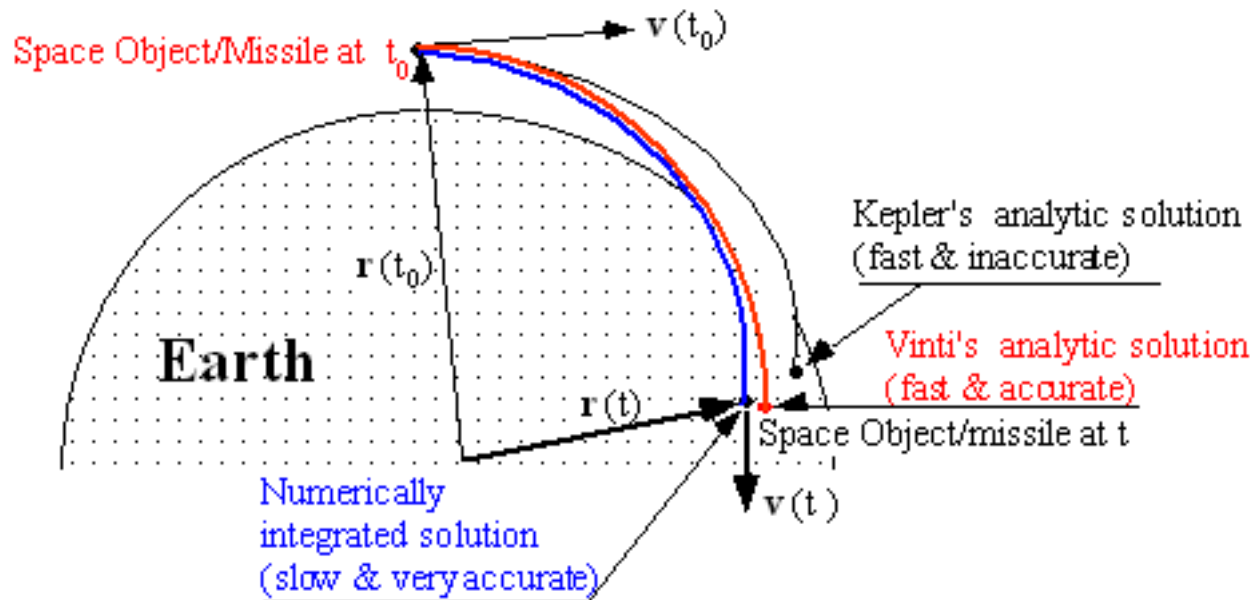
Kepler's Problem

Given: $\mathbf{r}(t_0), \mathbf{v}(t_0), t_0, t$

Find: $\mathbf{r}(t), \mathbf{v}(t)$

A matter of accuracy and CPU timing:

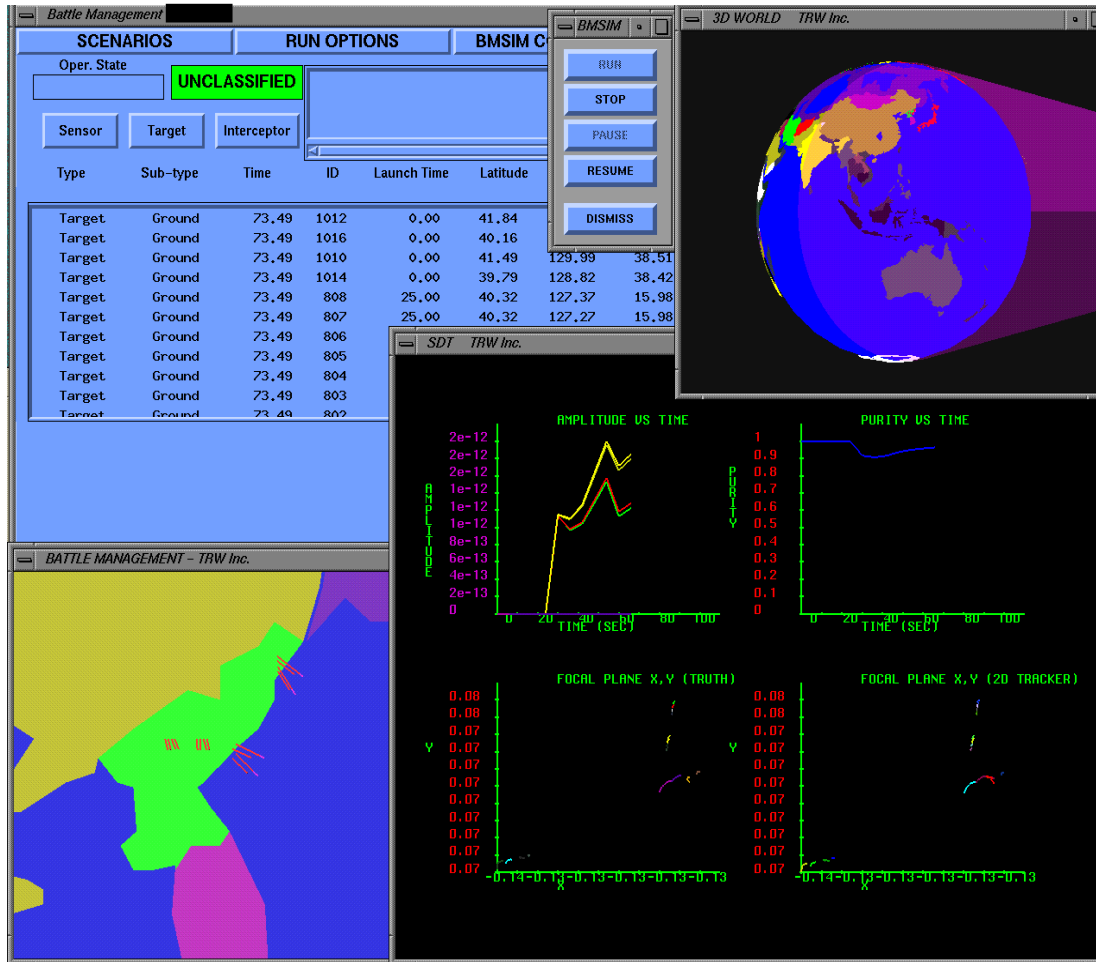
- Analytic Kepler (missile warning)
- **Analytic Vinti** (missile defense)
- **Numerical method** (missile warning)



There is a difference between **Missile Warning** and **Missile Defense**
Battle Management and Interceptor System require IPP analytic solutions

Simulation: Analytic and Numerical Methods

Battle Management (BM) WTA Application



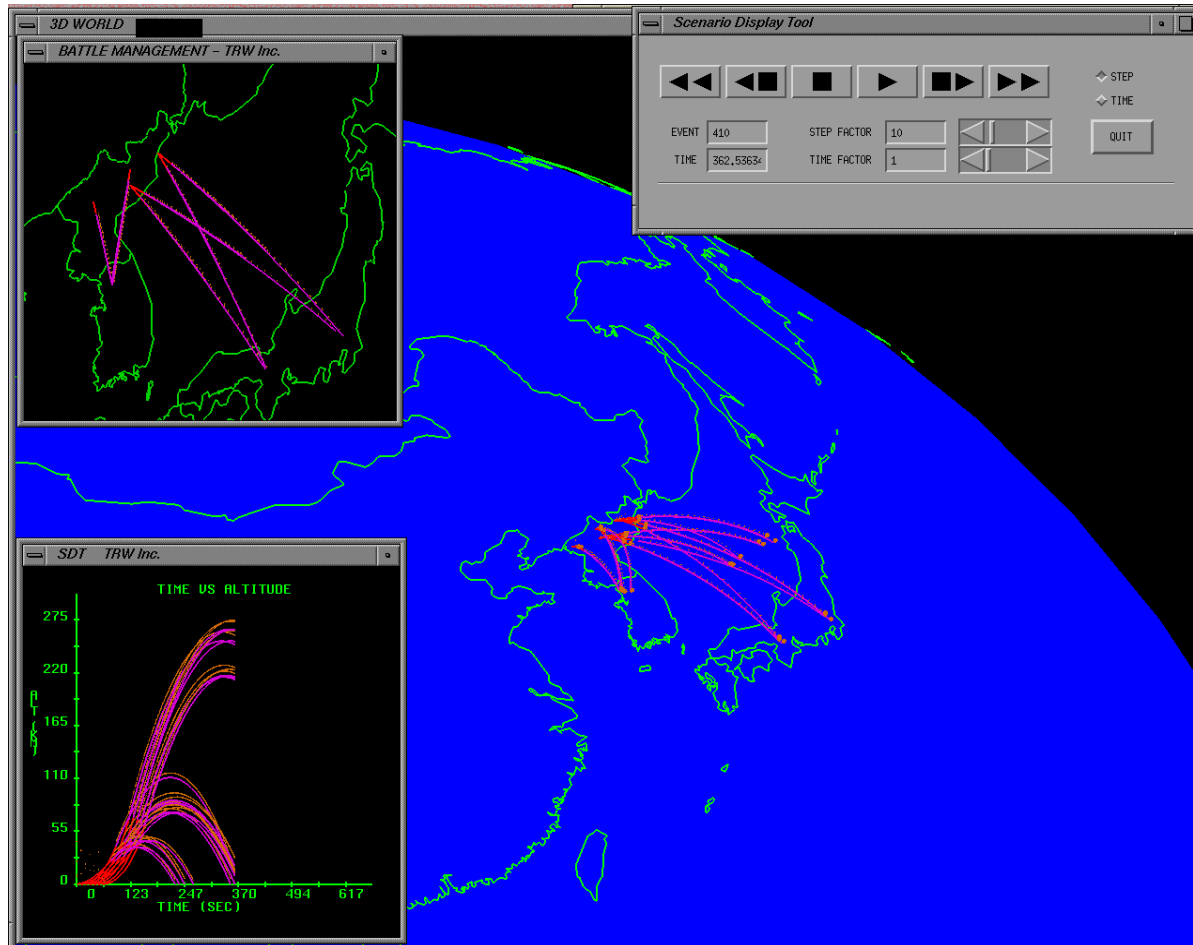
BM - Weapon
Target
Assignment
(Aircrafts/
Missiles/
Satellites)

- Only Vinti/Der's analytic trajectories fast and accurate
- Numerical trajectory computation too slow

Too Slow computing Numerical Trajectories for WTA

Simulation: Analytic and Numerical Methods

Multiple Missile IPPs Application



BM – Impact Point Predictions

- Only Vinti/Der's analytic trajectories fast and accurate
- Numerical trajectory computation too slow

A Few Minutes Too Late for any Intercept